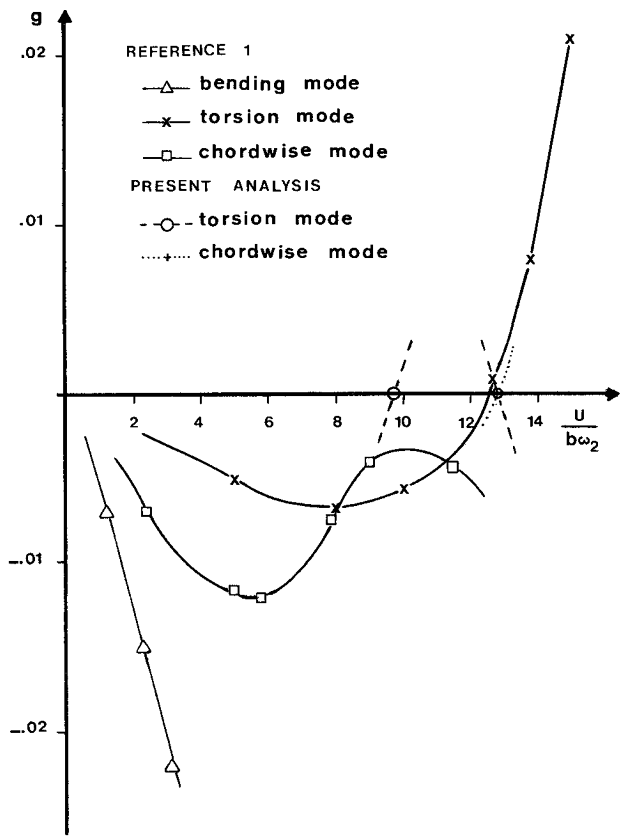

Fig. 2 Aeroelastic stability of a low aspect-ratio cantilever wing.

Hypersonic Flutter of a Cantilever Wing

Consider the hypersonic flutter of the cantilever wing studied in Ref. 7 and reported in Ref. 1. This model has been analyzed with great details in Ref. 4 using in-quadrature air loads. The pioneering and now classical work of Ref. 7, made well before the advent of high computational devices, was performed using the classical k method of flutter analysis. The main purpose of analyzing this example here is to show that the traditional V-g-ω plots using the k or p-k methods can miss some mild mode instabilities, since the solution is made at discreet values of reduced frequencies or velocities. The data of the model are given in the cited references. We consider only one case of Λ_3 (the ratio of chordwise to torsion frequency), namely, $\Lambda_3 = 1.833$, in this section. The results of the analysis are shown in Fig. 2. As can be seen from this figure, the problem presents two critical modes, the first one being a mild bending-torsion flutter mode, and the second being a violent torsion-chordwise flutter mode. The present formulation shows that the first instability occurs for the mild mode at a value of $U/b\omega_2 = 9.72$ and a flutter frequency ratio $\omega/\omega_2 = 0.7058$. This mode becomes stable again at a value of $U/b\omega_2 = 12.92$ and a frequency ratio of 0.643. The second mode becomes unstable at a value of $U/b\omega_2 = 12.97$ and a frequency ratio of 1.273. The analysis of Ref. 7 made for discrete values of reduced frequency jumps the first instability and only detects the second one.

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Prevention of Jump in Inertia-Coupled Roll Maneuvers of Aircraft

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Nomenclature

I_x, I_y, I_z	=	resp. roll, pitch, and yaw inertia moments
i_1, i_2, i_3	=	cyclically $(I_z - I_y)/I_x$, etc.
l, m, n	=	resp. roll, pitch, and yaw moment coefficients
p, q, r	=	resp. roll, pitch, and yaw rates
y, z	=	resp. side and normal force coefficients
α, β	=	resp. angle of attack and sideslip
$\delta a, \delta e, \delta r$	=	resp. aileron, elevator, and rudder deflection

Subscript

$\alpha, p, \ldots$	=	stability derivative w.r.t. $\alpha, p, \ldots$

Superscripts

^	=	indicates division by one of i_1, i_2, i_3
	=	indicates transpose

Introduction

THE problem of inertia coupling in rapidly rolling aircraft was first recognized when divergent motions were predicted by Phillips[1] at certain critical roll rates. These critical roll rates were obtained by Rhoads and Schuler[2] as steady-state solutions of simplified equations that neglected gravity terms. Such solutions obtained by neglecting the weight components in body axes are called pseudosteady state (PSS) solutions. The PSS method was employed by Schy and Hannah[3] for a coupled, nonlinear system of equations for the aircraft dynamics with a linear aerodynamic model to represent the equilibrium solutions as a function of a control parameter. Jump was shown to occur for a critical value of the control parameter at which a turning point (also termed a saddle-

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node bifurcation) was observed. Computer simulation of the nonlinear equations for selected control combinations was then used to accurately predict peak values.

The application of continuation methods by Carroll and Mehra[4] made it possible to calculate equilibrium solutions for a general nonlinear aerodynamic model. Bifurcation surfaces were used by Carroll and Mehra[4] to suggest possible nonlinear control interconnects between aileron and rudder which would avoid regions of jump in the state space. This Note describes a specific methodology to arrive at a *linear* interconnect relationship ($\delta r = k\delta a$) which avoids the saddle-node, thus preventing jump.

Pseudosteady State Analysis

The nonlinear equations of motion of Schy and Hannah[3] are adopted for this study with their linear aerodynamic model of aircraft B, data for which is obtained from Etkin.[5] These are a set of seven equations assuming constant speed and small angles of attack and sideslip. For the present study, this model is helpful since it isolates the nonlinear influence of the inertial coupling terms from the nonlinear aerodynamic terms which can vary widely from one aircraft to another. This model is expected to give a qualitatively correct picture of the bifurcation pattern and, as long as the saddle-node point occurs at small α and β, fairly accurate PSS solutions for jump onset.

By including terms nonlinear in α and β, one would be able to capture high α and high β PSS solutions accurately, and thus predict the magnitude of jump. Also, by introducing a nonlinear aerodynamic model in the equations for simulation, peak response values can be correctly achieved. Continuation techniques have recently been used by Jahnke and Culick[6] in such a study to determine pseudosteady states for roll-coupled maneuvers of aircraft. The approach presented in this Note remains equally applicable in such a general case.

For the model considered above, PSS solutions are obtained by solving

$$(pI - M)x = c + pb \tag{1}$$

$$l_\beta \beta + l_p p + l_q q - i_1 qr + l_{\delta a}\delta a + l_{\delta r}\delta r = 0 \tag{2}$$

where I is the identity matrix, and

$$M = \begin{bmatrix} 0 & \hat{n}_r & \hat{n}_\beta & 0 \\ -\hat{m}_q & 0 & 0 & -\hat{m}_\alpha \\ 1 & 0 & 0 & z_\alpha \\ 0 & 1 & -y_\beta & 0 \end{bmatrix}$$

$$x = (q, r, \beta, \alpha)'$$

$$c = (\hat{n}_{\delta a}\delta a + \hat{n}_{\delta r}\delta r, -\hat{m}_{\delta e}\delta e, z_{\delta e}\delta e, -y_{\delta a}\delta a - y_{\delta r}\delta r)'$$

$$b = (\hat{n}_p p, 0, 0, 0)'$$

The maneuver considered is a roll initiated from a trim α corresponding to a 2-deg elevator deflection. The turning point occurs for an aileron value of nearly -4 deg. It has been suggested in Ref. 3 that it might be possible to develop a criterion based on constraints on the values of the stability derivatives in the above equations that will ensure the existence of a single real solution for a given control parameter range. This possibility is first investigated.

A sensitivity analysis with all the stability derivatives used in the above formulation taken one at a time revealed n_β to be one of the most significant parameters in terms of its effect on the location of the turning point for the present data. PSS roll rate solutions for the basic aircraft and for three augmented values of n_β are shown in Fig. 1. It is seen that with increasing values of n_β, the turning point occurs for larger values of the control parameter (aileron deflection). Beyond a particular value of n_β, multiple solutions are no longer observed; but this requires n_β to be increased to about twice its original value. Such major changes, even if physically realistic, can rarely be accommodated in what must inevitably be an advanced stage in any design. Similar magnitudes of augmentation are found to be necessary for other parameters. Thus, the above suggestion must be discarded as impractical.

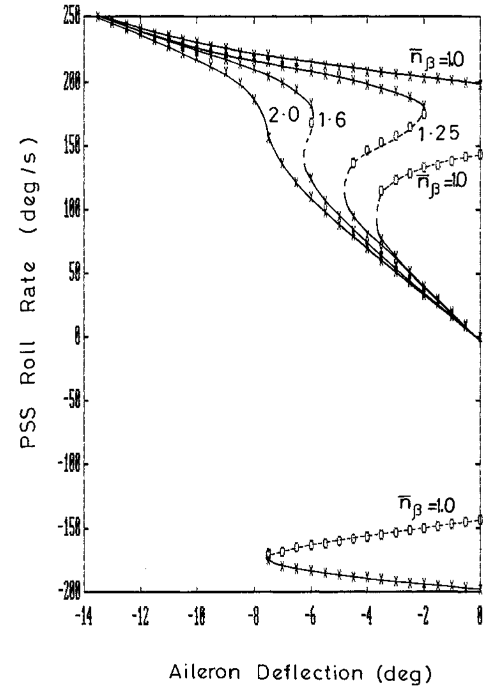

Fig. 1 PSS roll solutions for basic aircraft and for augmented $n_\beta = (\bar{n}_\beta)n_{\beta(\text{basic})}$. (Full lines are stable solutions, -o-o- are unstable divergent.)

Alternate Strategy

We select $n_{\delta a}$, the yawing moment caused by aileron deflection, as a candidate parameter instead. (The baseline dataset has $n_{\delta a} = 0$.) Defining an aileron-rudder interconnect by $\delta r = k\delta a$, the effective yaw control provided by the aileron is altered to $(n_{\delta a} + kn_{\delta r})\delta a$. We continue to refer to the bracketed quantity as (the augmented value of) $n_{\delta a}$. It is noticed that it is not possible to change the number and location of the five solutions for zero aileron using $n_{\delta a}$. It follows that multiple solutions are unavoidable.

Physically, $n_{\delta a}$ influences the yaw equilibrium and affects the sideslip developed in a rolling maneuver. It has been shown in Ref. 7 that even when the controls are withdrawn before the roll rate builds up to high values at jump, significant and rapid changes in sideslip occur which could lead to loss of control. In the present case, the negative trim α gets converted to negative β due to the rapid roll with the resulting proverse yaw speeding up the roll. Positive $n_{\delta a}$ values are expected to arrest the growth of sideslip as well as influence the nature of the PSS solutions.

An asymmetric point of bifurcation (see Huseyin[8]) is one at which two equilibrium paths intersect with an exchange of stabilities. A turning point is known to occur as a perturbation from an asymmetric point of bifurcation. Furthermore, a perturbation of opposite sign gives equilibrium paths that do not show a turning point. This leads us to investigate the possibility of identifying an asymmetric bifurcation point for our problem and of tuning a perturbation parameter to ensure that turning points are avoided.

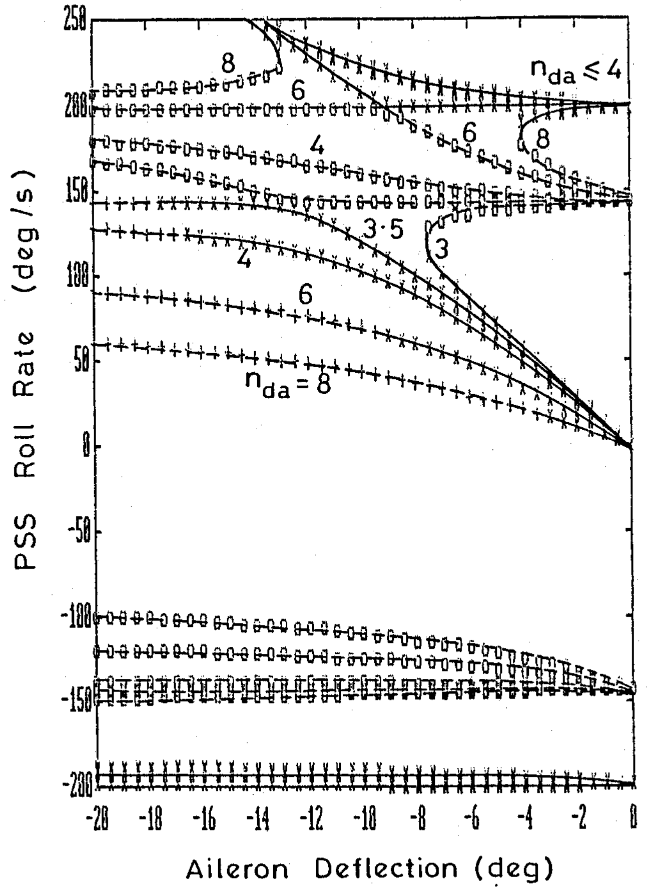

Fig. 2 PSS roll solutions for different values of $n_{\delta a}$. (Full lines are stable solutions, -o-o- are unstable divergent. -+-+- are unstable oscillatory.)

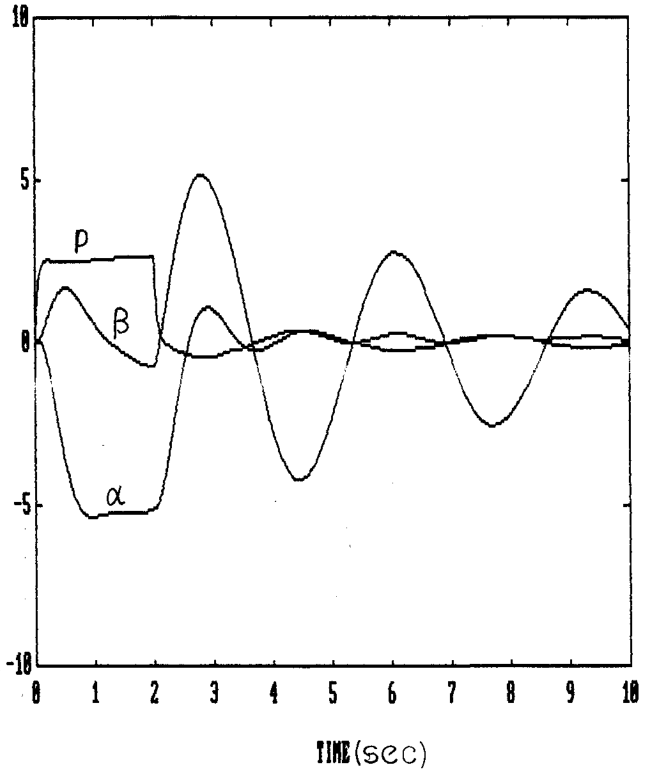

Fig. 3 Time histories of angles of attack and sideslip (deg), and roll rate (rad/s) for a 2-s pulse input of $\delta e = 2$ deg and $\delta a = -10$ deg with $n_{\delta a} = 4$.

Bifurcation maps for various values of $n_{\delta a}$ are plotted in Fig. 2. The lower limit of $n_{\delta a}$ is given by the (first) asymmetric bifurcation of the primary solution (branch with zero roll rate for zero δa) at an $n_{\delta a}$ value just under 3.5. For $n_{\delta a}$ values just below this, a turning point exists. For $n_{\delta a} = 4$, the primary solution continues until it loses stability to become oscillatory-divergent, a phenomenon known as Hopf bifurcation which leads to a family of limit cycles. With increasing values of $n_{\delta a}$, this transition to instability occurs for smaller values of aileron deflection. The upper limit on the value of $n_{\delta a}$ is given by the second asymmetric bifurcation (of a different equilibrium solution) at $n_{\delta a} = 6$. The nature of the bifurcation map for $n_{\delta a}$ values beyond 6 is illustrated in Fig. 2 by that for $n_{\delta a} = 8$. The early loss of stability and the possibility of unbounded divergence means that $n_{\delta a} > 6$ is undesirable. Thus, jump-free equilibrium can be achieved for $3.5 < n_{\delta a} < 6$.

Two things remain to be verified. Firstly, whether the $n_{\delta a}$ values obtained are realistic. In the present study, $n_{\delta a} = 4$ corresponds to an acceptable value of $k = 2.35$, which seems to suggest that aileron deflection is limited by maximum rudder availability. Secondly, are the α and β values over the allowable range of control deflections small enough to make the results reliable? This is best determined by computer simulations of the nonlinear equations since the PSS solutions do not provide any information about the transient response. Results of one such simulation for $n_{\delta a} = 4$ are shown in Fig. 3 where the time histories of roll rate, α and β are plotted over a time period of 10 s. The maneuver consists of a control combination of 2-deg elevator plus -10-deg aileron—both withdrawn after 2 s—designed to give a peak roll rate of about 150 deg/s. Peak values of angles of attack and sideslip are seen to be restricted to about 5 deg each. The simulation shows that in addition to providing the desired roll behavior, this technique also suppresses excessive sideslip, which agrees with the reasoning presented earlier in this Note.

Conclusions

The present study has outlined a method of avoiding jump behavior in inertia-coupled roll maneuvers of aircraft. It is found that a linear aileron-rudder interconnect relationship can be determined for this purpose. In order to evaluate our results, a simplified model from the literature has been used. The approach, however, remains equally valid for any complicated nonlinear set of equations of motion. The addition of nonlinear aerodynamic terms to the analysis is expected to provide better quantitative values while maintaining an unchanged qualitative picture.

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